assignment07

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**University of Southern California**  
**Marshall School of Business**  
**FBE 506 Quantitative Method in Finance**

Assignment 07  
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# **Question 1**

a11 <- 15  
a12 <- 1  
a21 <- 1  
a22 <- 2  
  
x1 <- .2  
x2 <- .5  
  
detA <- a11\*a22 - a12\*a21

## a. Find determinant of A

## b. Find the inverse of A

## c. Find the trace of A

## d. Find the quadractic form X’AX

## e. Find that min(X’AX)

Let . We have . Substitute in :

Find the critical point of x’Ax: The first order condition is:

Solving the two above equation together yields , and . Thus the point is the extrema.

To test for the second order condition, form the Hessian:

We can see:

Thus H is positive definite and have a minimum at (0,0,0).

## f. Find that min(X’AX) s.t.

Similar to e. but adding the constraint

The Lagrange becomes:

Find the critical point of The first order condition is:

Also

Solving these yields: , and

The bordered Hessian matrix is:

Thus the min x’Ax happens at

## g. Find the eigenvalues of A.

We have the 2x2 system where

Solve the characteristic equation of this system for the eigenvalues:

## h. Find the eigenvectors of A.

For :  
We need to solve:

By inspection, the notrivial solutions to this system and hence the eigenvectors for the eigenvalue are all nonzero scalar multiples of:

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# **Question 2**

## a. Find ,

## b. Find

The matrix of cofactors from C is:

Thus, the adjoint of C is:

Assume C is invertible then:

## c. Find AB’, A’B, CA’, BCA’

## d. Find tr(C)

## e.Find a vector orthogonal to A

V is orthogonal to A if V.A = 0. Thus, find value for V that satisfy:

From observation, we can see there is a V(1,1,-3) satisfy the condition.

## f. Find the eigenvalues of A.

The characteristic polynomial of C is:

The eigenvalues therefore must satisfy the cubic equation:

Verify:

C <- matrix(c(1,3,-1,2,3,1,-1,1,0),3,3, byrow=TRUE)  
ev <- eigen(C)  
ev$values

## [1] 4.658587 -1.757702 1.099115

## g. Find the eigenvectors of C.

Using R, we see the corresponding eigenvectors:

ev$vectors

## [,1] [,2] [,3]  
## [1,] 0.62852346 0.6713753 -0.6035847  
## [2,] 0.77713615 -0.4117461 0.2341098  
## [3,] 0.03190081 0.6162145 0.7621536

# **Question 3**

## a. Find the GMV solution, no short sale.

Given parameters

# Mean Returns  
mean\_rStock1 <- .023  
mean\_rStock2 <- .021  
mean\_rf <- .008  
  
# Variance of returns  
var\_rStock1 <- .0072  
var\_rStock2 <- .0049  
  
# Standard deviation of returns  
sd\_rStock1 <- sqrt(var\_rStock1)  
sd\_rStock2 <- sqrt(var\_rStock2)  
  
# Covariance of Returns  
cov\_rStock1\_rStock2 <- .0023  
  
# Correlation of Returns  
cor\_rStock2\_rStock2 <- cov\_rStock1\_rStock2 / (sd\_rStock1 \* sd\_rStock2)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Asset | Mean Returns | Variance | Pair |  |
| Stock 1 | 0.023 | 0.0072 | (1,2) | 0.0023 |
| Stock 2 | 0.021 | 0.0049 |  |  |
| Risk Free | 0.008 |  |  |  |

Since the investor’s objective is to minimize risk, we solve for the global minimum variance combination of the two assets. Let denotes the weight of the investment in asset i , and assume all money is invested in i, meaning .

The global minimum variance portfolio is the solution of the constrained minimization problem:

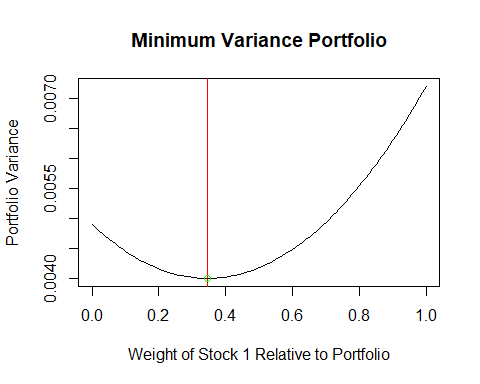
Find the critical point:

**Recap:**

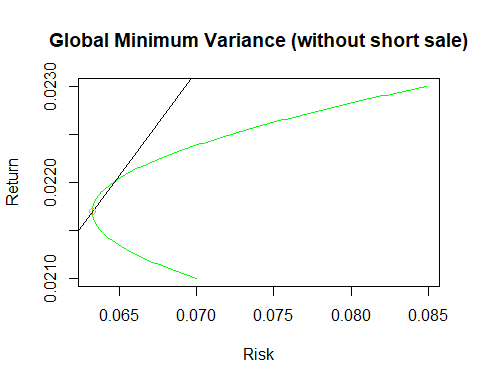
|  |  |
| --- | --- |
| Asset | Weight |
| Stock 1 | 34.67% |
| Stock 2 | 65.33% |

Graphing the function:

# The Variance function of the portfolio  
fVar <- function(m1) var\_rStock1 \* m1^2 + var\_rStock2 \* (1 - m1)^2 + 2 \* m1 \* (1 - m1) \* cov\_rStock1\_rStock2  
  
# Set the weighting  
m1 <- .3467  
m2 <- .6533  
  
# Plot of the variance function with respect to w1.  
w1 <- seq(0, 1, .05)  
plot(fVar, w1, xlab = "Weight of Stock 1 Relative to Portfolio", ylab = "Portfolio Variance", main = "Minimum Variance Portfolio"); abline(v = m1, col = "red")  
points(m1, fVar(m1), col='green')



# Plot of the efficient frontier, assume no short sale allowed  
w1WithoutShort <- seq(0, 1, .005)  
fVar <- function(w1) var\_rStock1 \* w1^2 + var\_rStock2 \* (1 - w1)^2 + 2 \* w1 \* (1 - w1) \* cov\_rStock1\_rStock2  
r <- function(w1) mean\_rStock1\*w1 + mean\_rStock2\*(1-w1)  
riskWithoutShort <- sqrt(fVar(w1WithoutShort))  
returnWithoutShort <- r(w1WithoutShort)  
plot(riskWithoutShort, returnWithoutShort, type='l', main='Global Minimum Variance (without short sale)', xlab = 'Risk', ylab = 'Return', col = 'green')  
wp <- (var\_rStock2 - cov\_rStock1\_rStock2) / (var\_rStock1 + var\_rStock2 - 2 \* cov\_rStock1\_rStock2) # Normalizing weight  
Rp <- r(wp)  
sigma <- sqrt(fVar(wp))  
Rf <- mean\_rf  
Sharpe <- (Rp-Rf)/sigma  
X <- seq(0, 1, .01)  
Y <- Rf + Sharpe\*X  
lines(X, Y)  
points(sigma, Rp, col='orange')



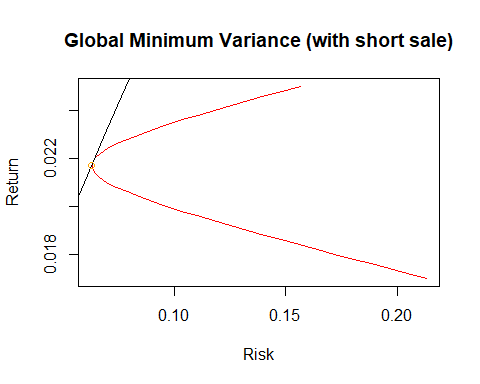
We see indeed the minimum is at

Thus, the optimum weights of each stocks in the portfolio to satisfy the objective of minimum risk is:

|  |  |  |  |
| --- | --- | --- | --- |
| Asset | Weight | Risk | Return |
| Stock 1 | 34.67% |  |  |
| Stock 2 | 65.33% |  |  |
| Portfolio | 100% | 0.063235 | 0.0216933 |

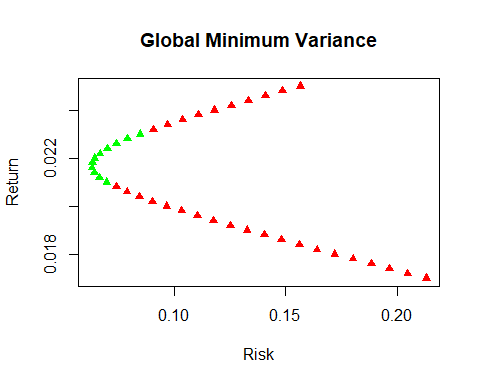
## **b. Construct the efficient frontier. Assume short sale.**

# Plot of the efficient frontier, assume short sale allowed  
w1WithShort <- seq(-2, 2, .1)  
riskWithShort <- sqrt(fVar(w1WithShort))  
returnWithShort <- r(w1WithShort)  
plot(riskWithShort, returnWithShort, type='l', main='Global Minimum Variance (with short sale)', xlab = 'Risk', ylab = 'Return', col = 'red')  
wp <- (var\_rStock2 - cov\_rStock1\_rStock2) / (var\_rStock1 + var\_rStock2 - 2 \* cov\_rStock1\_rStock2) # Normalizing weight  
Rp <- r(wp)  
sigma <- sqrt(fVar(wp))  
Rf <- mean\_rf  
Sharpe <- (Rp-Rf)/sigma  
X <- seq(0, 1, .01)  
Y <- Rf + Sharpe\*X  
lines(X, Y)  
points(sigma, Rp, col='orange')



## c. **Graph the efficient frontier of portfolio with and without short sale together.**

# Changed input to display less points  
w1WithoutShort <- seq(0, 1, .1)  
riskWithoutShort <- sqrt(fVar(w1WithoutShort))  
returnWithoutShort <- r(w1WithoutShort)  
w1WithShort <- seq(-2, 2, .1)  
riskWithShort <- sqrt(fVar(w1WithShort))  
returnWithShort <- r(w1WithShort)  
  
  
plot(riskWithShort, returnWithShort, main='Global Minimum Variance', xlab = 'Risk', ylab = 'Return', col = 'red', pch = 17)  
points(riskWithoutShort, returnWithoutShort, col = 'green', pch = 17)



## d. **Graph the CAL and find the point of tangency.**

w1 <- .3467  
Rp <- r(w1)  
sigma <- sqrt(fVar(w1))  
CAL\_slope <-(Rp - mean\_rf) / sigma  
CAL <- function(x) {mean\_rf + CAL\_slope \* x}

The portfolio returns is:

The portfolio returns variance is:

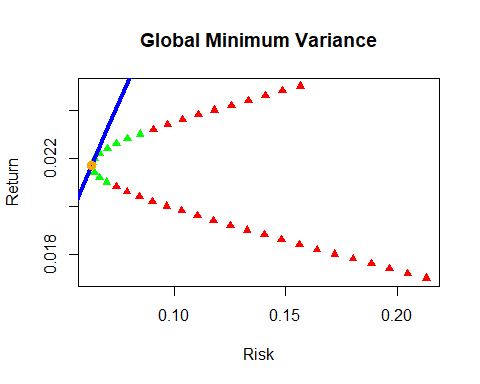
The slope of the Capital Allocation Line is:

**Thus the Capital Allocation Line equation is .**

**The tangency point is where the optimum weight is, i.e.  or**

Graphing

# Plot of the efficient frontier, assume no short sale allowed  
plot(riskWithShort, returnWithShort, main='Global Minimum Variance', xlab = 'Risk', ylab = 'Return', col = 'red', pch = 17)  
points(riskWithoutShort, returnWithoutShort, col = 'green', pch = 17)  
plot.function(CAL, add= TRUE, col = 'blue', lwd = 4)  
points(sigma, Rp, col='orange', lwd = 4)



# **Question 4**

## a. Find the GMV solution.

library(matlib)  
  
vcov <- matrix(c(.04, .02, .001, .02, .03, .01, .001, .01, .025), 3, 3, byrow=TRUE)  
vcovInverse <- inv(vcov)  
vcovInverse

## [,1] [,2] [,3]  
## [1,] 39.70678 -29.93280 10.38485  
## [2,] -29.93280 61.02627 -23.21319  
## [3,] 10.38485 -23.21319 48.86988

Formulating the Markowitz portfolio problem:

Let denotes a target expected return level. Formulate the problem:

To solve this, form the Lagrangian function:

Because there are two constraints ( and ) there are two Langrange multipliers and . The first order condition for a minimum are the linear equations:

Simplify, we have:

Rewrite in matrix form:

or

The solution for is:

topMatrix <- cbind(2\*vcovInverse, rep(1, 3))  
bottomVector <- c(rep(1, 3), 0)  
AmMatrix <- rbind(topMatrix, bottomVector)  
bVector <- c(rep(0, 3), 1)  
zmMatrix <- solve(AmMatrix)%\*%bVector  
mVector <- zmMatrix[1:3,1]  
mVector

## [1] 0.3885350 0.3821656 0.2292994

Thus, the optimum weights of each stocks in the portfolio to satisfy the objective of minimum risk is:

|  |  |
| --- | --- |
| Asset | Weight |
| Stock A | 38.8535% |
| Stock B | 38.21656% |
| Stock C | 22.92994% |

## b. Find the GMV solution with constraint.

vcovInverse

## [,1] [,2] [,3]  
## [1,] 39.70678 -29.93280 10.38485  
## [2,] -29.93280 61.02627 -23.21319  
## [3,] 10.38485 -23.21319 48.86988

mean\_rStockA <- .0375  
mean\_rStockB <- .0420  
mean\_rStockC <- .0300  
mean\_rf <- .0225  
returnsVector <- rbind(mean\_rStockA, mean\_rStockB, mean\_rStockC) # Establish returns vector  
riskPremiumMatrix <- returnsVector - mean\_rf

We have the z vector:

z <- vcovInverse %\*% riskPremiumMatrix

The lambda value is thus:

lambda <- sum(z)  
w1 <- z[1] / lambda  
w2 <- z[2] / lambda  
w3 <- z[3] / lambda

Therefore, the weight of these stocks are:

**Recap:**

|  |  |
| --- | --- |
| Asset | Weight |
| Stock A | 12.3628107% |
| Stock B | 78.0497035% |
| Stock C | 9.5874858% |